

Given an array A, let's say that there is an inversion between indices i and j if $i < j$ but $A[i] > A[j]$.

What is the array containing the elements {1, 2, 3, 4, 5, 6} that has the maximum number of inversions?

{ 1, 2, 3, 4, 5, 6 }

{ 1, 2, 3, 4, 5, 6 }

$i < j$ 2 < 3 ✓

$A[i] > A[j]$ 2 > 3 ✗

How many inversions does the array from the previous question have?

{ 6, 5, 4, 3, 2, 1 }

$i < j$ $A[i] > A[j]$

1 < 2 } 6 > 5

1 < 3 } 6 > 4

1 < 4 } 6 > 3

1 < 5 } 6 > 2

5

1 < 6, 6 > 1

2 < 3, 5 > 4

2 < 4, 5 > 3 etc....

Pattern

5 + 4 + 3 + 2 + 1

15

What is the most number of inversions possible for an array of n elements?

$$\begin{matrix} n-1 & n \\ 5 & \times & 6 = 30 \end{matrix}$$

$$\frac{30}{2} = 15$$

$$\frac{n(n-1)}{2}$$